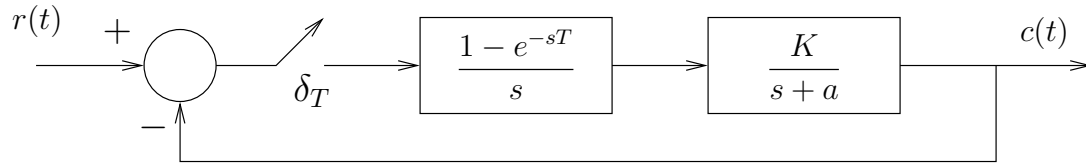
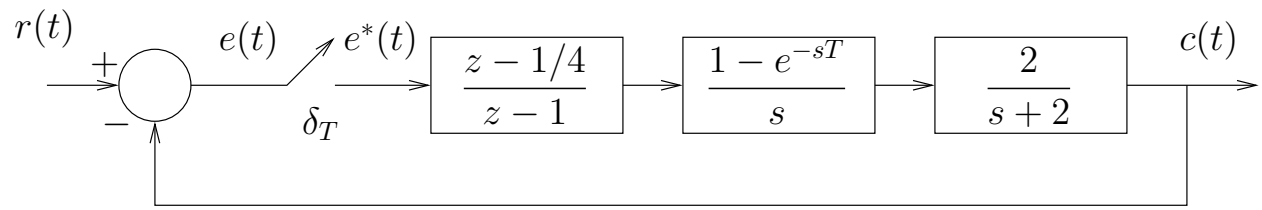


Q.1 For what values of the gain  $K$  is this closed-loop system stable:



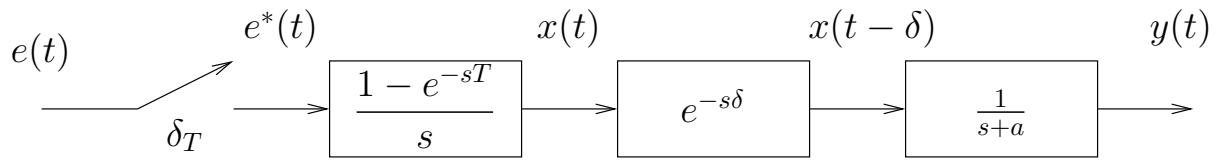
Leave your answer in terms of  $a$  and the sampling period:  $T$ .

Q.2 Consider the following closed-loop system:

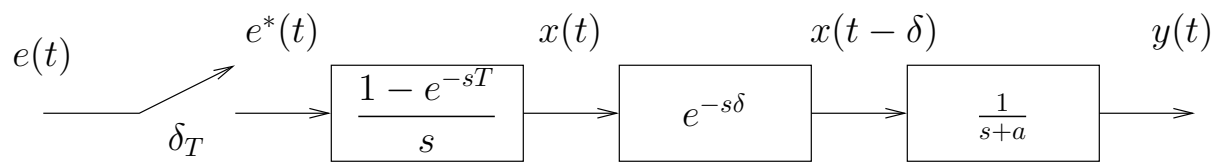


Compute the discrete-time output,  $y(kT)$ , in response to a unit step input. Let  $T = \ln(2)$ .

**Q.3** The “First-Order plus Dead Time” plant is one of the most common systems encountered in chemical process control. Consider the following block diagram of an ideal sample and hold, followed by a delay of  $\delta$  seconds, followed by a first-order system:



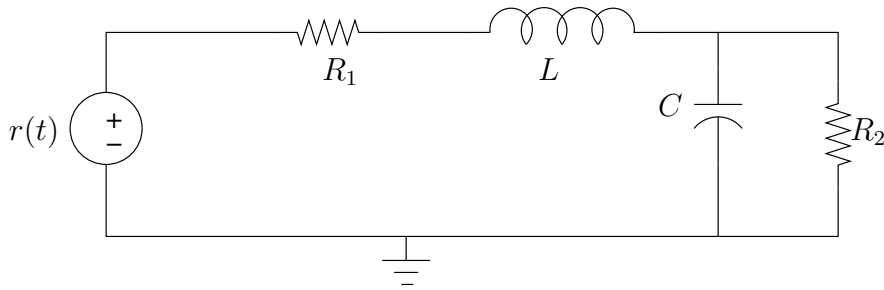
- (a) Determine the pulse transfer function,  $\frac{Y(z)}{E(z)}$ , between the samples of  $e(t)$  and the samples of  $y(t)$ .



(b) Determine the **continuous-time** impulse response,  $g_D(t)$ , between  $e^*(t)$  and  $y(t)$ .

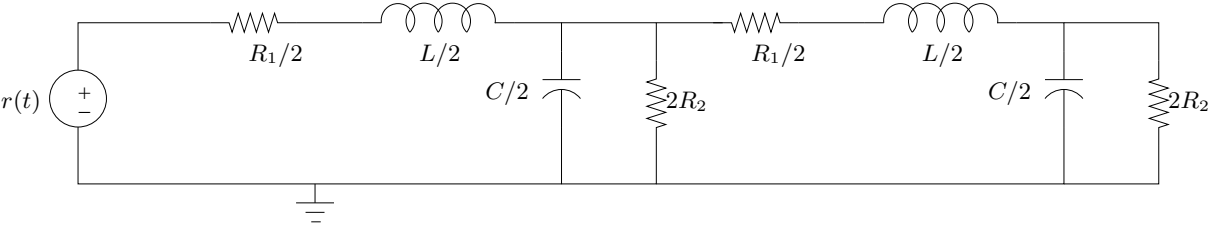
- (c) Use the time-shifting property of the  $Z$  transform to check your answer to part (a), above, for the special case where the delay  $\delta$  is an **integer** number of time steps,  $\delta = nT$ , for  $n = 0, 1, 2, \dots$

- Q.4** (a) Construct a continuous-time state-space model for the following RLC circuit. Let the output be the voltage across  $R_2$ .

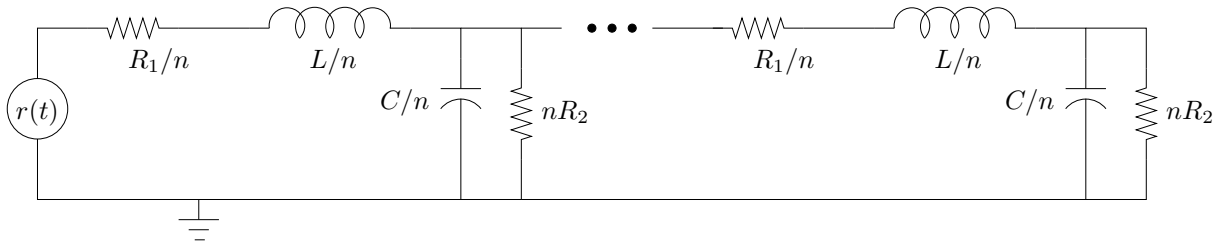


*Hint: Use the capacitor charge and inductor current as your states. Remember that the voltage across an inductor is given by  $v(t) = Ldi(t)/dt$ , and that the capacitor voltage is  $v(t) = q(t)/C$ .*

(b) Suppose that two such LRC circuits are joined together, as follows. Let the output be the voltage drop across the rightmost resistor. Construct a state-space model for this system:



- (c) Suppose that  $n$  such LRC circuits are joined together, as follows. Let the output be the voltage drop across the rightmost resistor. Construct a state-space model for this system:

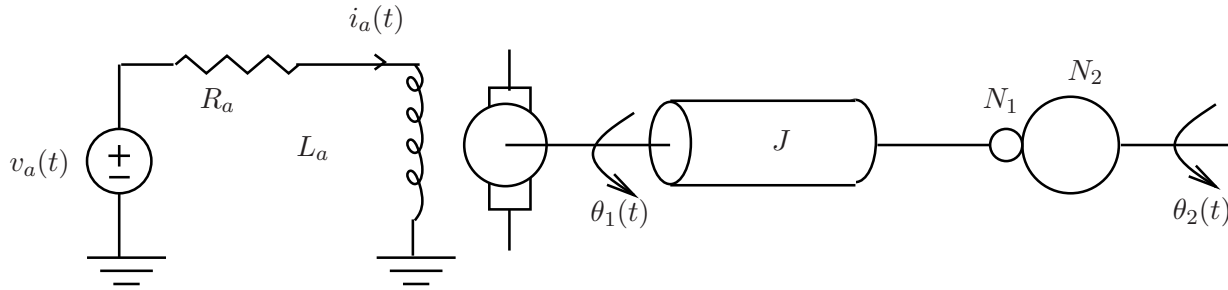




**Q.5** Consider the armature-controlled electric motor shown below with back e.m.f.  $v_b = -K_b\omega_1(t)$  where  $\omega_1$  is motor shaft angular velocity. The input voltage comes from a control based on the shaft position before the gear

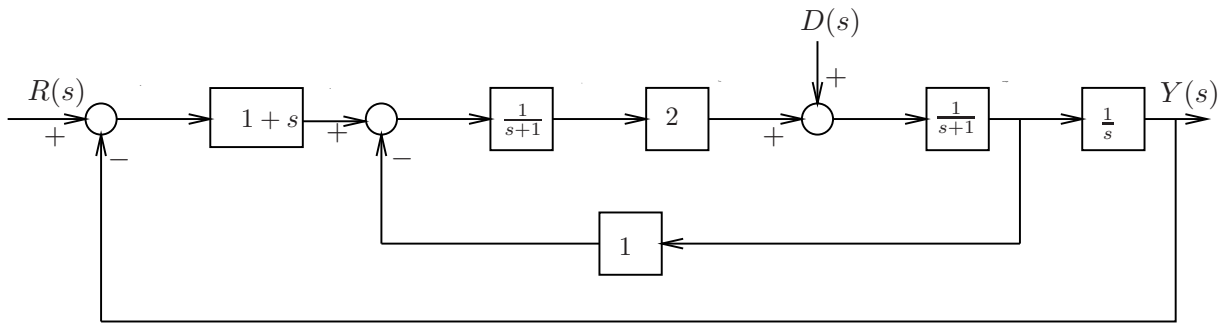
$$v_a(t) = K_c(\theta_{\text{des}}(t) - \theta_1(t)) + K_I \int (\theta_{\text{des}}(t) - \theta_1(t)) dt$$

Denote the armature resistance and inductance as  $R_a$  and  $L_a$ . Denote the mechanical inertia and friction coefficient as  $J$  and  $D$ . The gear ratio is  $N_2/N_1$ . The motor constant is  $K_m$ .



Draw the block diagram with input  $\theta_{\text{des}}(s)$  and output  $\theta_2(s)$ , with separate blocks for control, electrical dynamics, motor constant, mechanical dynamics, gears, and back e.m.f. Make sure you write the correct transfer functions in all blocks. Label all signals.

Q.6

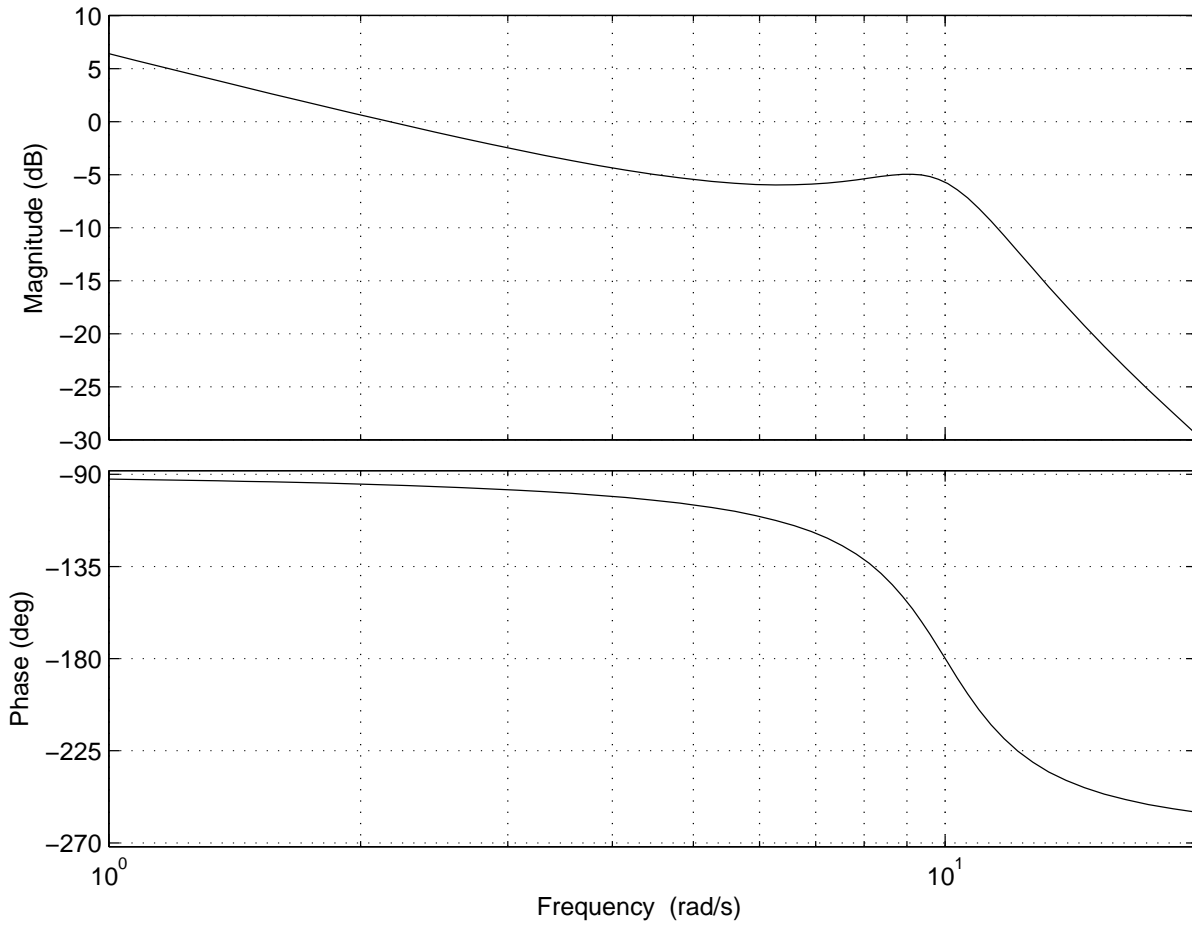


Find the transfer function  $Y(s)/D(s)$  using block-diagram rearrangement and reduction techniques. There are no marks for only doing algebra, even if you get the right answer.

Q.7

Consider the open-loop transfer function's Bode diagram shown below. Note that at  $\omega = 9$  rad/s the gain is  $-5$ dB and the phase is  $-152^\circ$ .

Bode Diagram



For the multiple choice questions, circle the choice that is closest to the correct answer. Sketch the Nyquist diagram in one of the blank plots on the following page. (If you use both, clearly indicate which one you want marked.)

What is the closed-loop system's phase margin?

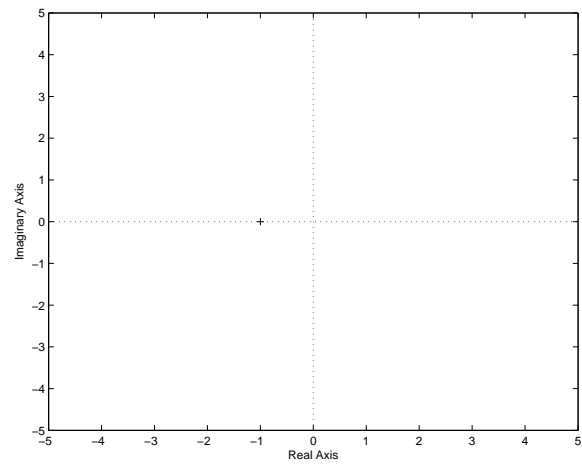
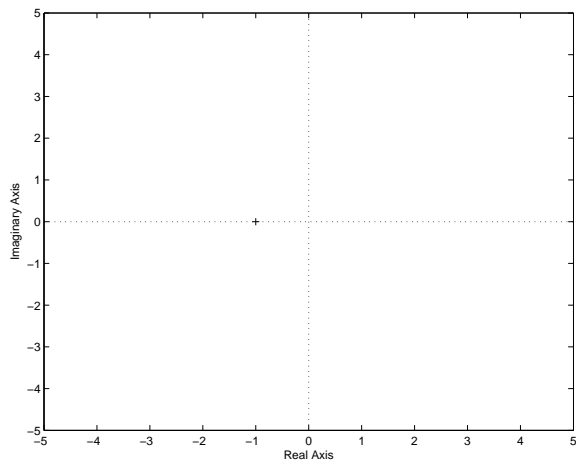
- a) none (unstable)      b)  $85^\circ$       c)  $28^\circ$       d) infinity

What is the closed-loop system's gain margin?

- a) 1.5      b) -6 dB      c) 6 dB      d) infinity

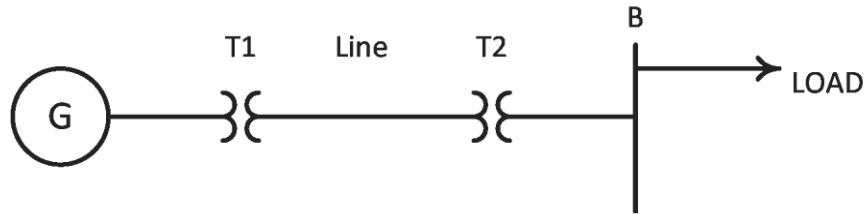
What is the closed-loop system's steady-state error for a ramp input  $r(t) = t$ ?

- a) 0      b) 0.5      c) 10      d) infinity



TOPIC: Power transmission (ENEL487)

Consider the single-line diagram shown below:



- G is a 25MVA 11kV generator with negligible internal impedance
  - T1 is a 25 MVA 11kV : 138 kV transformer with per-unit impedance  $Z_1 = 0.01 + j0.03$  .
  - The transmission line between T1 and T2 is rated at 138kV, 25MVA, with per unit impedance  $Z_{Line} = 0.005 + j0.05$
  - T2 is a 20 MVA 138kV:13.6kV transformer with actual reactance of  $X_2 = j0.37\Omega$  when viewed from the low voltage side
  - Assume Bus B to be at 1.0pu voltage
1. The Load is rated at 20MVA
    - a. Draw the single line per-unit diagram, calculating per-unit impedances for a system with  $S_{Base} = 25\text{MVA}$
    - b. Calculate the actual generator terminal voltage when the Load draws rated current at unity power factor
  2. The load requirement is increased by 40%
    - a. Calculate the voltage at each end of the transmission line
    - b. Calculate the actual losses in the system

Topic: Power Electronics (ENEL585)

Q1. A 3-phase 6-pulse diode rectifier employs an LC filter with an inductance of 100mH and capacitance of 770 $\mu$ F. The input is an ideal 1100V 60Hz ac source. The load is a pure resistance of 33 $\Omega$ .

Note: When asked to "Explain" that means please provide numerical values to illustrate your point.

- (a) Draw a circuit diagram and label important voltages and currents.
- (b) Carefully sketch (show max and min values) line-to-line voltage, inductor current, output voltage, line current waveforms. Find average output current. Note: The output current is thru the pure resistance.
- (c) Showing steps clearly, calculate the input power factor from information in part (b).
- (d) If the capacitor fails as an open circuit, what is the effect on input PF and output voltage. Explain.  
To be clear: The circuit operates with an inductor of 100mH and the capacitor is removed.
- (e) If the inductor fails as a short circuit, what is the effect on input PF and output voltage. Explain.  
To be clear: Circuit operates with  $C = 770\mu\text{F}$ , the inductor is shorted, and each diode conducts  $21.27^\circ$ .
- (f) For part (e) sketch: line-to-line voltage,  $v_{AB}$ , the output voltage,  $v_R$ , and line current  $i_A$ , one beneath the other (to show the phase relationship) and indicate which diodes are conducting in each time instant.

**TOPIC: Power System Economics (ENEL587, ENEL597)**

Q1. Two utilities establish an economic energy exchange initiative to reduce their costs and improve the reliability of their systems. Utility A has a composite cost function of  $C(P_A) = 10 + 5P_A + 0.30P_A^2$  \$/h, and utility B has a composite cost function of  $C(P_B) = 10 + 6P_B + 0.35P_B^2$  \$/h. In area A, local demand is 150 MW and available local generation is between 20 and 200 MW. In area B, local demand is 200 MW and local generation is between 30d and 250 MW. The area is interconnected by a transmission line.

- . a) Assuming transmission cost is negligible with a maximum capacity of 50 MW, formulate a optimal dispatch optimization problem to minimize the cost of joint operation. Clearly write the objective function and all the required constraints.
- . b) Find the optimal dispatch of the areas by forming the Lagrangian function and writing the KKT optimality conditions for the problem you formulated in part a. Also find the system marginal cost.
- . c) Calculate the payment of the receiving utility to the exporting utility based on the split-the-savings- equally criterion.
- . d) Assume the transmission business in the area is transferred to an independent company who asks a flat rate of 10.2 \$/MWh for any transaction over the line. Reformulate the optimization problem of part a and find the new optimal dispatch.

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Field of Study Examination, Feb 24 2017

Subject area: Control Systems and Power

This question paper has 13 pages (not including this cover page).

This question paper has 6 questions.

Answer a minimum of one question and at most three questions from this subject area.

If you attempt questions 1-3 (Control Systems) on pp. 1-9 from this exam, please do so in the space provided on question paper itself, fill in your UCID and name and hand it in. Do not hand in a blue answer booklet.

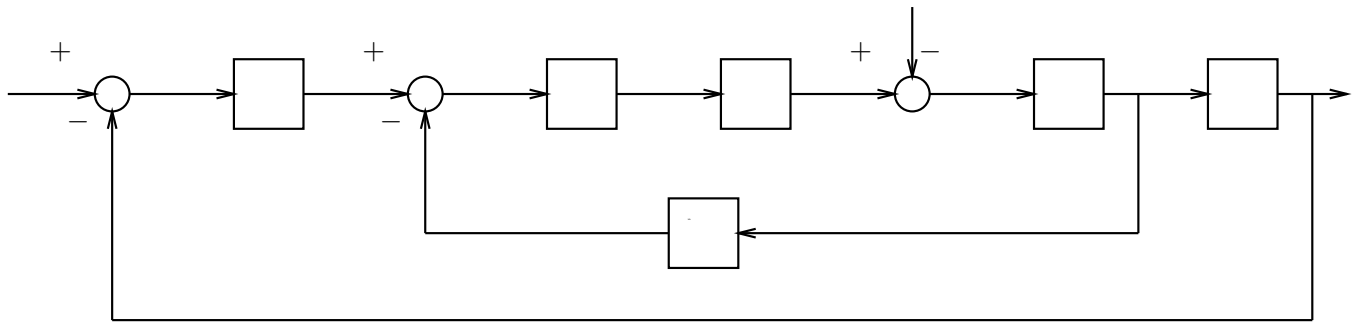
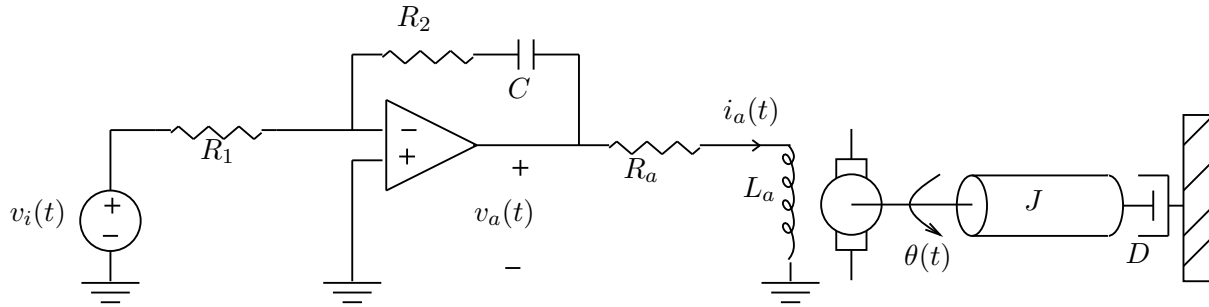
If you attempt questions 4-6 (Power) on pp. 10-13 from this exam, please use a separate booklet (i.e., blue booklet) for answering them.

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1. This question must be answered on the question paper and handed in.

An armature-controlled electric motor with ideal op-amp control and its block diagram is shown below. The input voltage to the op-amp is  $v_i(t) = -e(t) = -(\theta_{\text{des}}(t) - \theta(t))$



Put the following labels on the block diagram in the correct places.

signal	description	transfer function	description
$\theta(s)$	angular position		
$\theta_{\text{des}}(s)$	desired angle	$G_c(s)$	negative of op-amp transfer function $-V_a(s)/V_i(s)$
$T_m(s)$	motor torque	$\frac{1}{s}$	integrator
$V_a(s)$	armature voltage	$K_m$	motor constant $T_m(s)/I_a(s)$
$E(s)$	error $\theta_{\text{des}}(s) - \theta(s)$	$G_a(s)$	electric circuit transfer function with output $I_a(s)$
$I_a(s)$	armature current	$G_l(s)$	mechanical transfer function $\omega(s)/T_l(s)$
$T_d(s)$	disturbance torque	$K_b$	back e.m.f. constant for the motor $V_b(s)/\omega(s)$
$T_l(s)$	total torque on load		
$\omega(s)$	angular velocity		

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2. This question has 3 parts (a)-(c). This question must be answered on the question paper and handed in.

Consider the transfer function for a plant:

$$G_p(s) = \frac{10(s - 0.9)}{(s + 1)(s^2 + 2s + 100)}$$

with a PID controller

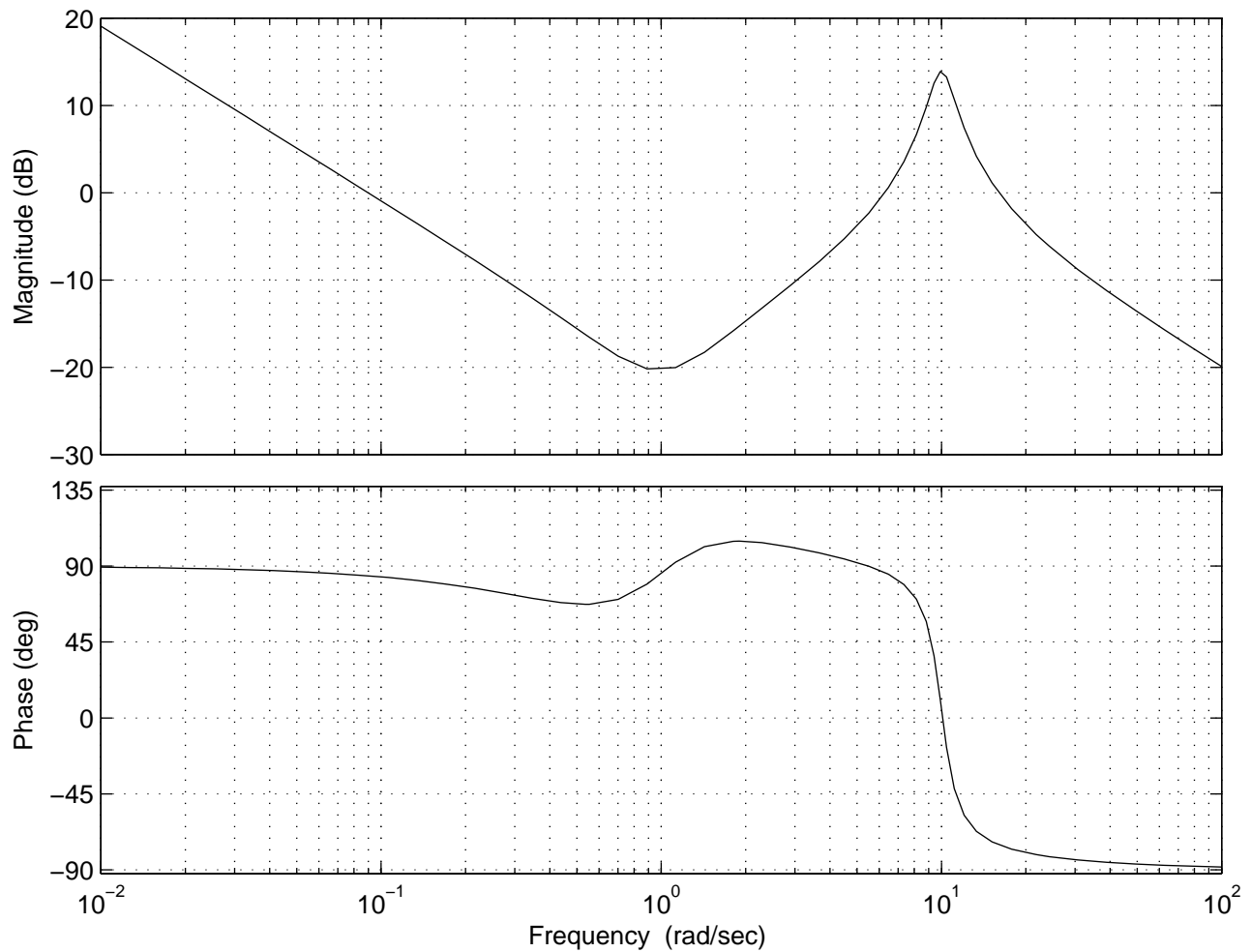
$$G_c(s) = 1 + s + \frac{1}{s}$$

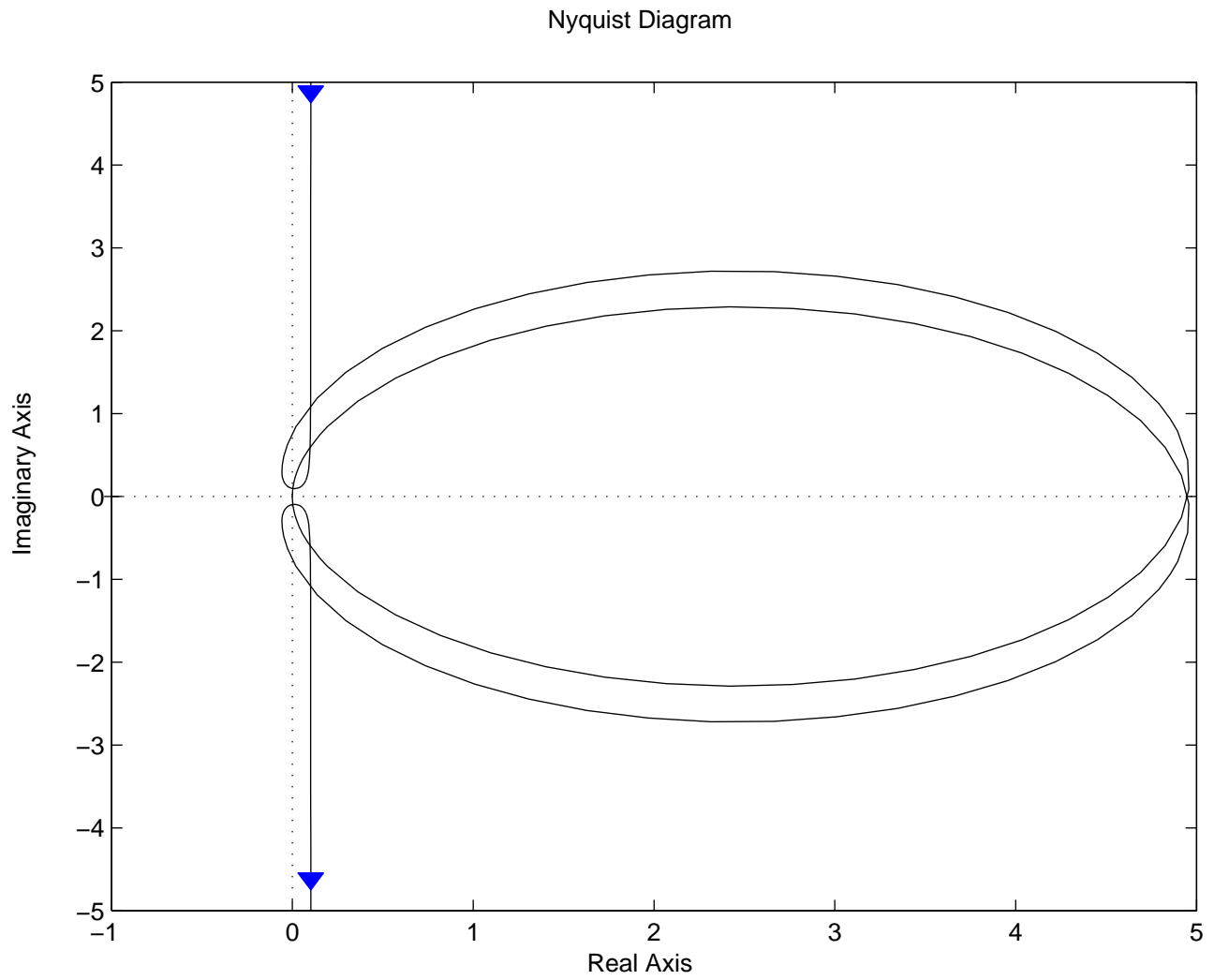
giving total open-loop transfer function

$$G_c(s)G_p(s) = \frac{(s^2 + s + 1)}{s} \frac{10(s - 0.9)}{(s + 1)(s^2 + 2s + 100)}$$

The Bode Diagram and Nyquist Plot for the open-loop transfer function are given to you:

Bode Diagram





- (a) Determine whether the closed-loop transfer function  $G_c G_p / (G_c G_p + 1)$  is stable or unstable.
- (b) If the closed-loop system is stable estimate the gain margin and phase margin.
- (c) If the closed-loop system is unstable determine how many closed loop poles are in the right-half plane.

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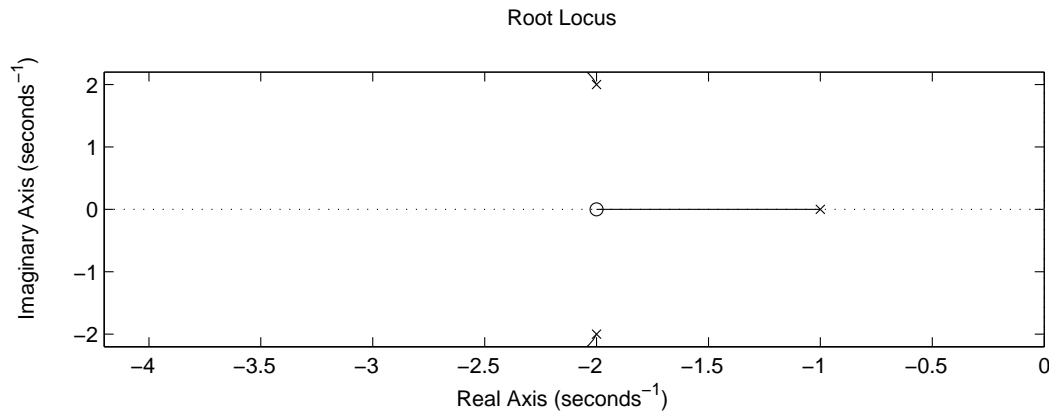
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3. This question has 5 parts (a)-(e). They are multiple choice questions with options A-D which must be marked on the question paper and handed in.

Consider the root locus of the open-loop transfer function:

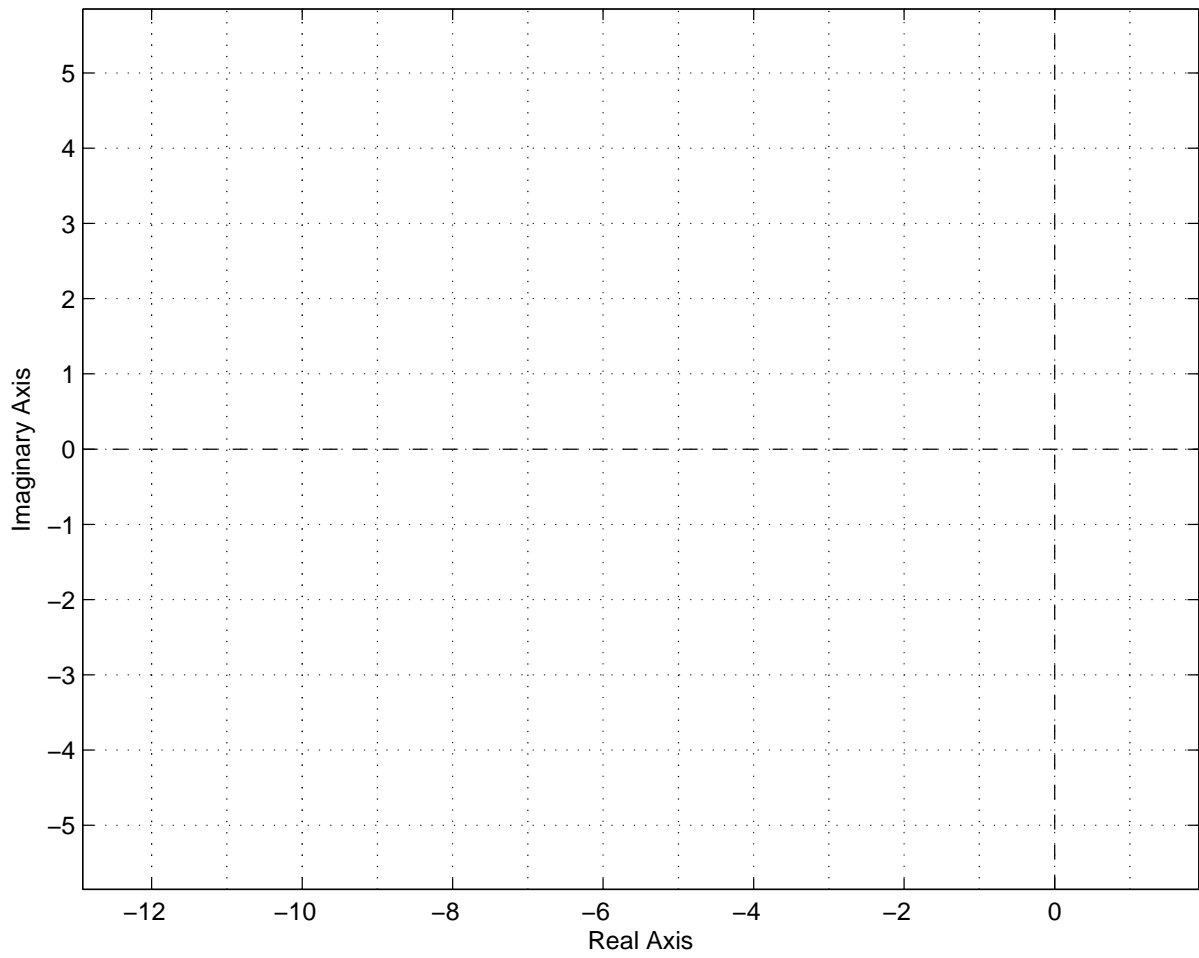
$$G(s) = \frac{K(s+5)(s+2)}{(s+1)(s^2+4s+8)}$$

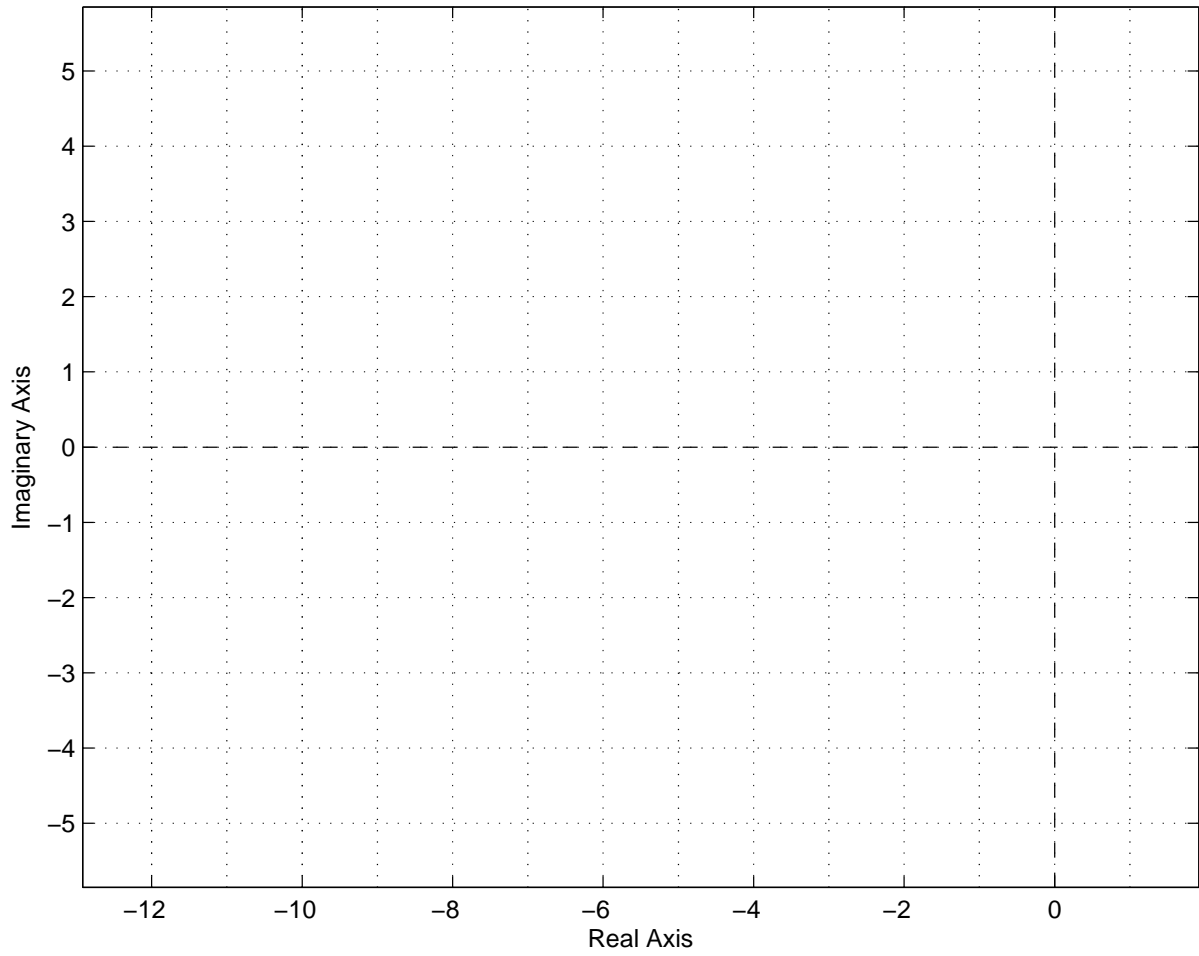


Some of the root locus is displayed above. You can draw the whole thing yourself on the next page.

For some of the following questions you will want to use the blank root locus plots on the following pages, that have equal axes and 1 cm spacing, to figure things out. Circle the choice that is closest to the real answer.

- (a) Where are the asymptotes ?  
**A)**  $540^\circ$       **B)**  $\pm 90^\circ$  centred at 2      **C)**  $-180^\circ, \pm 60^\circ$  centred at 2      **D)** no asymptotes
- (b) What is the angle of departure from the pole at  $-2 + 2j$ ?  
**A)**  $87^\circ$       **B)**  $97^\circ$       **C)**  $117^\circ$       **D)**  $137^\circ$
- (c) What is the point of arrival on the real axis?  
**A)** 2.4873      **B)** -4.6335      **C)** -9.1095      **D)** no point of arrival
- (d) What is the gain K when the system goes unstable?  
**A)** 10      **B)** 20      **C)** 40      **D)** It will never go unstable
- (e) What is the gain K when a closed-loop pole exists at -1.5?  
**A)** 0      **B)** 1.25      **C)** 8.25      **D)** not possible







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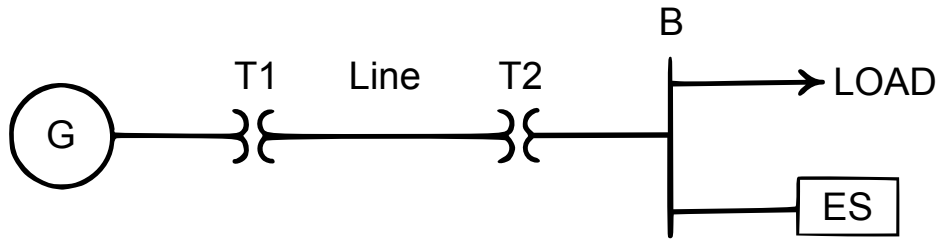
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4. This question has 5 parts (a)-(e). This question must be answered in a blue answer booklet.

Consider the single line diagram shown below



- G is a 50 MVA 13.2 kV Generator with negligible internal impedance
  - T1 is a 50 MVA, 13.2 kV:69 kV Transformer with  $j0.1$  per-unit impedance
  - The Line between T1 and T2 is rated at 50 MVA, 69 kV and has impedance  $0.3 + j3$
  - T2 is a 45 MVA 69 kV:11 kV Transformer with  $j0.08$  per-unit impedance
  - The Load operates at 30 MW with power factor 0.8 lagging
  - ES is an energy storage facility
  - Assume Bus B to be at  $1.0\angle 0^\circ$  per-unit voltage
- (a) Calculate the per-unit impedance of the Line and T2
- (b) Calculate the per unit current at the load, and the corresponding RMS actual generator voltage when there is no power flow at the energy storage facility
- (c) What is the maximum per-unit current flow through the line if the system is to operate within the component ratings?
- (d) Calculate the actual power flow into ES if the Line current is at the maximum permitted and the current at ES is at unity power factor
- (e) Calculate the power factor at the Line into B if ES is supplying 5 MW at unity power factor

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5. This question has 6 parts (a)-(f). This question must be answered in a blue answer booklet.

A 3-phase 6-pulse diode rectifier employs an LC filter with an inductance of 25 mH and capacitance of 3500  $\mu\text{F}$ . The input is an ideal 600 V 60 Hz ac source. The load is a pure resistance of 10  $\Omega$ .

- (a) Draw a circuit diagram and label important voltages and currents. Indicate standard diode numbering.
- (b) Showing min and max, sketch: line-to-line voltages, line current (neglect ripple), bridge voltage, inductor current (with ripple), output voltage across the  $R$  (neglect ripple), output current through the  $R$  (neglect ripple). Note: for the sketches, show the approximate phase relationship between inductor current and bridge rectified voltage. Calculate average output voltage and current. Find the inductor current peak-to-peak ripple, and the output voltage peak-to-peak ripple.
- (c) Calculate the input power factor based on information in part (b).
- (d) Sketch line-to-line voltage  $v_{AB}$  and diode 1 forward voltage (ie,  $v_{AK1}$ ). What would be a reasonable Peak Inverse Voltage rating (ie, specification) for each diode (assume a safety reliability factor of 2)?
- (e) If the inductor fails as a short circuit, what is the effect on input PF and average output voltage (for this RC case)? Note: diode conduction angle is now  $18.7^\circ$ . Provide numerical values.
- (f) For part (e) sketch, showing min and max values: line-to-line voltage  $v_{AB}$ , the output voltage  $v_R$ , and line current  $i_A$ , one beneath the other (**show the phase relationship, indicate which diodes are conducting**).

6. This question has 2 parts (a)-(b). Part (a) has 6 subparts (i)-(vi) and part (b) has 2 subparts (i)-(ii). This question must be answered in a blue answer booklet.

(a) The cost functions for three power plants are given below.

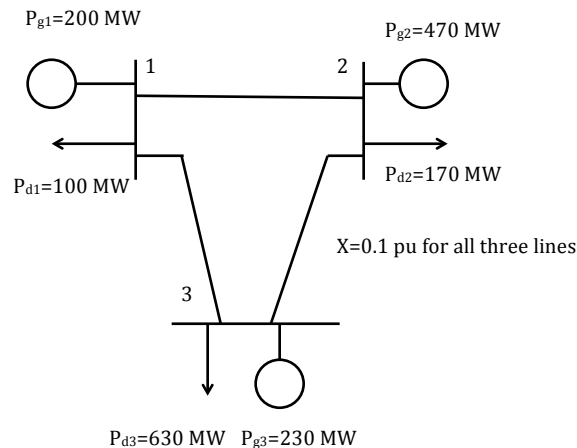
$$\begin{aligned}
 C_1(P_1) &= 210 + 7.1P_1 + 0.00217P_1^2 \quad (\$/h) & 30 \leq P_1 \leq 300 \quad (MW) \\
 C_2(P_2) &= 770 + 6.1P_2 + 0.00201P_2^2 \quad (\$/h) & 30 \leq P_2 \leq 550 \quad (MW) \\
 C_3(P_3) &= 300 + 6.7P_3 + 0.00278P_3^2 \quad (\$/h) & 30 \leq P_3 \leq 550 \quad (MW)
 \end{aligned}$$

The system operator aims at finding the cheapest generation dispatch schedule for the next time interval when the system demand is expected to be 900 MW.

- i. Formulate a cost-minimization problem for the above system, include the objective and all necessary constraints.
- ii. Using the formulation from part (i), create a Lagrangian function when ignoring all inequality constraints.
- iii. Apply the KKT Conditions of Optimality to the Lagrangian function of part (ii) and find the optimal output of the three generators as well as the system marginal cost.
- iv. Check your solution against the inequality constraints and modify the solution if necessary.
- v. Calculate the total generation cost.
- vi. If the demand increases by 2 MW, how much would the estimated total system cost be. Do not recalculate the costs. Only use the values that you found in previous steps.

(b) The generators #1, #2 and #3 in (a) are in fact located on three different buses, and are scheduled to produce 200 MW, 470 MW and 230 MW, respectively, as shown in the figure below. The load is also distributed among the three buses and is also shown on the figure. The reactances for the three lossless lines are identical and equal to  $x = 0.1$  pu on a base of 100 MVA.

- i. Using the DC-power flow method, find the bus angles in degrees. Bus #1 is the slack bus with  $V_1 = 1$  and  $\delta_1 = 0^\circ$ .
- ii. Using the angle values of part (i), find the line flows for  $P_{21}$ ,  $P_{13}$  and  $P_{23}$ .



(.. continued on next page)

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You may use the following:

DC Power Flow:

$$[B][\delta] = [P] \quad p_{ij} = \frac{1}{x_{ij}}(\delta_i - \delta_j)$$

$B$  (for DC power flow): -(the imaginary part of the  $Y_{bus}$  matrix, built when the resistive elements are ignored, tap values set to 1, reactor elements ignored. Also the row and column associated with the slack bus are ignored.)