

TOPIC: Electrostatics and Magnetostatics E EL475

Q.1

We are interested in finding the potential $V(\vec{\mathbf{r}})$ at a point on the z -axis $P = (0, 0, z)$ for a uniform charge density ρ_S distributed on a disk of radius $r = a$ lying in the xy -plane and centred around the origin, assuming the reference is chosen as infinity.

(a) Draw a clear sketch of this problem to analyze the geometry. Your sketch should clearly indicate the field point P , the field position vector $\vec{\mathbf{r}}$, and the source position vector $\vec{\mathbf{r}}'$.

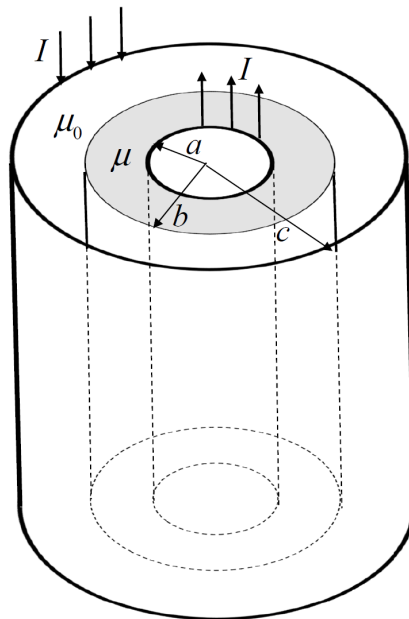
(b) Solve for the potential $V(\vec{\mathbf{r}})$ at a point on the z -axis $P = (0, 0, z)$

Note: You will probably require one of the following integrals in order to solve this problem

$$\begin{aligned} \int \frac{1}{\sqrt{x^2 + a^2}} dx &= \ln(x + \sqrt{x^2 + a^2}) & \int \frac{x}{\sqrt{x^2 + a^2}} dx &= \sqrt{x^2 + a^2} \\ \int \frac{1}{x^2 + a^2} dx &= \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) & \int \frac{x}{x^2 + a^2} dx &= \frac{1}{2} \ln(x^2 + a^2) \end{aligned}$$

Q.2

An infinitely long coaxial cable has a *hollow* inner conductor of radius a , which has a cladding of radius b which is a magnetic material of permeability μ , as depicted in the figure below. The inner conductor carries the *total* supply current I , and the outer conductor carries the *total* return current I ; both are distributed uniformly around their respective cylindrical surface in the directions indicated in the figure.



- (a) Choose an appropriate coordinate system and, *using Ampere's law*, show that the magnetic field everywhere in the region $a < \rho < b$ is given by

$$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi \frac{\text{A}}{\text{m}}$$

- (b) Using the given field in part (a), calculate the stored magnetic energy in the cladding region for a length d of the cable and for a current I .
- (c) If the relative permeability of the cladding is $\mu_r = 20$, what is the inductance of a length d of this cable?

Q.3

Starting from Gauss' law in integral form, produce a derivation of the concept of Divergence by showing that

$$\nabla \cdot \vec{D} = \lim_{\Delta v \rightarrow 0} \frac{\oint_S \vec{D} \cdot d\vec{S}}{\Delta v}$$

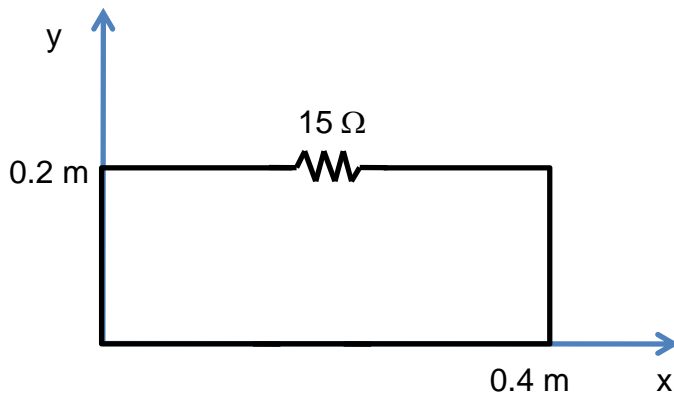
Q.4

Starting from a simple microscopic/atomic basis, explain how each of the following macroscopic properties is manifest:

- (a) Conductivity σ
- (b) Permittivity ε
- (c) Permeability μ

Q.1 Consider the loop containing a resistor as shown below. The loop is placed in a magnetic flux density described by:

$$\mathbf{B} = -20 \cos(100\pi t - \pi/3) \mathbf{a}_z \text{ mWb/m}^2$$



- Find the EMF (V_{emf})
- Calculate the induced current in the loop. Indicate the direction of current flow during the first quarter period on the figure above.

Q.2 A ground penetrating radar system is modeled as a uniform plane wave in free space impinging on the ground at normal incidence. The incident electric field (in free space, so properties are ϵ_0 , μ_0 , $\sigma=0$) is given by:

$$\mathbf{E}^i(x,t) = 10 \cos(10^9 t - 3.3x) \mathbf{a}_y \text{ V/m}$$

- Find the wavelength.

The ground has properties of $\epsilon_r=4$, $\mu_r=1$, and $\sigma=0.1 \text{ S/m}$.

- Calculate the reflection (Γ) and transmission (T) coefficients.
- Find an expression for the reflected electric field ($\mathbf{E}^r(x,t)$).
- Find an expression for the transmitted electric ($\mathbf{E}^t(x,t)$) and magnetic fields ($\mathbf{H}^t(x,t)$).

Q.3 A distortionless transmission line has $R=5 \Omega/\text{m}$, $L=20 \mu\text{H}/\text{m}$, $C=30 \text{nF}/\text{m}$ and is operated at 10 MHz. Calculate the following quantities:

- a) G
- b) the impedance of the line, Z_o
- c) the attenuation of the line, α
- d) the phase constant of the line, β
- e) the wavelength on the line, λ

A load of $Z_L=30-j40 \Omega$ is attached to 6 cm of the transmission line.

- f) Using the appropriate equation, find Z_{in} for the section of transmission line terminated by the load.

Q.4 A load of impedance $Z_L=30-j60 \Omega$ is attached to a transmission line with 75Ω characteristic impedance ($Z_o=75 \Omega$). The frequency of operation is 5 GHz and the wavelength on the line is 6 cm. Use the Smith Chart to solve the following questions.

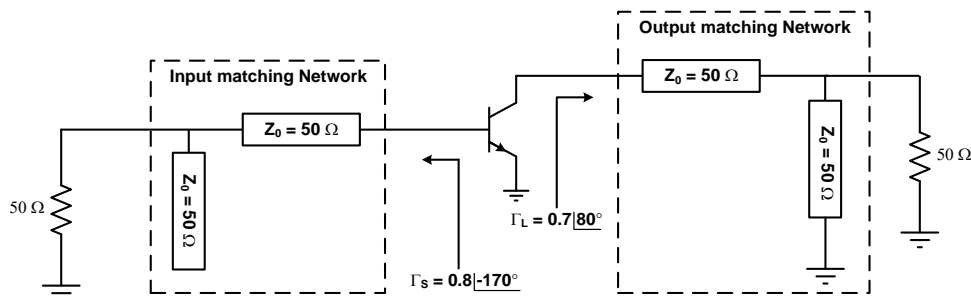
- a) Find the reflection coefficient at the load (Γ).
- b) Find the standing wave ratio (s). Verify with the appropriate equation
- c) Find the input impedance Z_{in} for a line of length of 5.0 cm attached to the load.
- d) Find the admittance of the load (Y_L).
- e) Find the distance from the load to the first voltage minimum.
- f) Find the shortest distance to a purely resistive load.

Q.5 A load of impedance $Z_L=70+j25 \Omega$ is attached to a transmission line with 100Ω characteristic impedance ($Z_o=100 \Omega$). The frequency of operation is 900 MHz and the wavelength on the line is 67 cm. To match the load to the line, design a series stub tuner with an open termination on the stub, and a shunt stub tuner with a short termination on the stub.

TOPIC: Microwave Active Circuits E EL574

Q.1 Determine the S parameters of two port network consisting of a series resistance R terminated at its input and output ports by the characteristic impedance Z_0 .

Q.2



- Design the input matching network by giving the lengths of 50 Ohms transmission lines as function of the wavelength, λ as shown in the above figure to produce the source reflection coefficient $\Gamma_s = 0.8 @ -170^\circ$ at 1 GHz.
- Design the output matching network by giving the lengths of 50 Ohms transmission lines as function of the wavelength, λ as shown in the above figure to produce the load reflection coefficient $\Gamma_L = 0.7 @ 80^\circ$ at 1 GHz.

Q.3

An amplifier is driven by modulated signal having a 20 MHz bandwidth is constituted by the cascade of three amplifiers A1, A2 and A3 and having the following characteristics:

Amplifier A1: $G_1=23$ dB, $IP3_{-1}=23$ dBm, $NF_1=2$ dB.

Amplifier A2: $G_2=17$ dB, $IP3_{-2}=42$ dBm, $NF_2=3$ dB.

Amplifier A3: $G_3=13$ dB, $IP3_{-3}=50$ dBm, $NF_3=5$ dB.

- The parameters $IP3_N$ are specified at the output of each amplifier and the reference noise temperature T_o is 290°K.
- Determine the equivalent noise figure NF_e of the power amplifier as well as its equivalent noise temperature T_e .
- Calculate the third order interception point IP_3 at the output of the power amplifier. Deduce the P_{1dB} of the power amplifier.

- d) Determine the carrier to third order intermodulation products ratio (C/IMD_3) at the output the power amplifier when it is driven with a two-tone signal spaced by 10 MHz and having a total input power of -8 dBm

Q.4 The scattering and noise parameters of a GaAs FET transistor at 2 GHz are:

$$S_{11} = 0.9 \angle -60^\circ, S_{21} = 3.1 \angle 140^\circ, S_{12} = 0.02 \angle 62^\circ \text{ and } S_{22} = 0.8 \angle -27^\circ$$

$$F_{\min} = 1.5 \text{ dB}, \Gamma_{\text{opt}} = 0.7 \angle 55^\circ, r_n = 0.95$$

- Study the stability of the device and draw the input and output stability circles in the Smith Chart,
- Can the device be considered unilateral?
- Draw the operating power gain circle for $G_p = 20 \text{ dB}$.
- Determine the source and load reflection coefficients required to design an amplifier to have an operating power gain of 20 dB. Explain your choices of Γ_s and Γ_L .

Q.1 An amplifier is attached to an antenna through a 9 cm long section of a coaxial cable. Assume the coax line has no loss, and its parameters are $C=96$ pF/m and $L=240$ nH/m. The antenna has a radiation impedance of 50Ω (you can assume the antenna is your generator, and its radiation impedance the generator internal impedance), however the amplifier is not matched to the system. Measurements showed that the amp has an input impedance of $(25+j20) \Omega$. The system operates at 10 GHz.

You have been asked to improve the system performance. Here are your tasks:

1. Assume that the antenna generates voltage V_g . Compute the voltage and the current at the input terminal of the transmission line (so from the antenna side, remember, your antenna acts as a generator here), find out the amplitudes of forward and backward travelling waves.
2. Compute available power. What percentage of available power at the generator is delivered to the amp?
3. Now match the system using lumped elements. What percentage of available power is delivered to the amp?

Remember to calculate the actual values of necessary inductances or capacitances.

- Q.2 Consider a rectangular waveguide with the following dimensions: $a=22.0\text{mm}$, $b=10.0\text{mm}$. The waveguide walls are made of a perfect conductor, waveguide is filled with air (lossless). Frequency is 10 GHz
- How much the TE_{20} mode will be attenuated at that frequency over a distance of 1 cm (expressed in dB)
 - The waveguide is loaded with an impedance of $(154.6+j154.6)\Omega$. Match the system using double stub serial tuner. The tuners are short-circuited, and separated by $5/4\lambda$ (lambda). You are allowed to add 1 or 2 sections of $\lambda/8$ section of a waveguide between the tuner and the impedance (only if you find it necessary). Draw the structure, pay attention to the feasibility of your structure (that is, could you actually build it? Show all dimensions (in mm). In particular, show how the serial connections of the waveguide and the tuners look like.

Q.3

Design a three-port resistive divider for an equal power split and 75 ohm system impedance (lets' call this system A). Derive S^A matrix of this system. Now terminate port 3 of the 3-port system A with a 150 ohm resistor - effectively turning the initial 3 port structure into a 2 port structure with matrix S^B . Compute S^B_{21} in this new system.

