TOPIC: Probability and statistics (ENEL419)

Q.1

(a)	A certain company regularly orders computers from three companies $(A, B, \text{and } C)$. From past experience it is known that 20% of the computers ordered are from company A, 50% are from company B and 30% are from company C. It is also known from experience that 5% of computers from company A are faulty, 4% of computers from company B are faulty and 8% of computers from company C are faulty. (i) Let F represent the event that there will be a faulty computer in an order. What is the probability that there will be a faulty computer? (ii) Suppose a computer is randomly selected from a batch and is found to be defective. Which company is it most likely to have come from and why?
(b)	The probability that a doctor correctly diagnoses a particular illness (event D) is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient sues is 0.9. Denote event "patient sues" as F. What is the probability that the doctor makes an incorrect diagnosis and the patient sues.

Q.2

(a)	Prove that $P\left[\overline{A} \cap \overline{B}\right] = 1 + P[A \cap B] - P[A] - P[B]$
(b)	Prove that $P\left[\overline{A} \cup \overline{B}\right] = 1 - P[A \cap B]$
(c)	Suppose a shelf contains 5 novels, 3 books of poem and a dictionary. (i) An experiment is performed where three books are picked at random from the shelf. What is the population and what is the sample for this experiment? (ii) Three books are selected at random. What is the probability that 2 are novels and 1 is a book of poem? (iii) Suppose only one book is selected at random. What is the probability that it is a dictionary?

Q.3 The voltage output, X, of a random voltage generator can assume values from zero to 1 volt with the following probability density function:

$$f(x)=\begin{cases}2(1-x), & 0\le x\le 1\\ 0, & \text{elsewhere}\end{cases}$$

(i) Find and sketch the cumulative distribution function $F(x)$. Indicate all the ranges of the variable x from $-\infty$ to $+\infty$ for $F(x)$. Also, you must label the axis.

(ii) Find the probability $P\left[|X-0.25|\le 0.5\right]$

(iii) Suppose the voltage, X, is applied to the input of a nonlinear device with input/output characteristic $Y=X^2+X+1$. What is the average value of the output voltage, Y?

(iv) What is the standard deviation of the voltage, X?

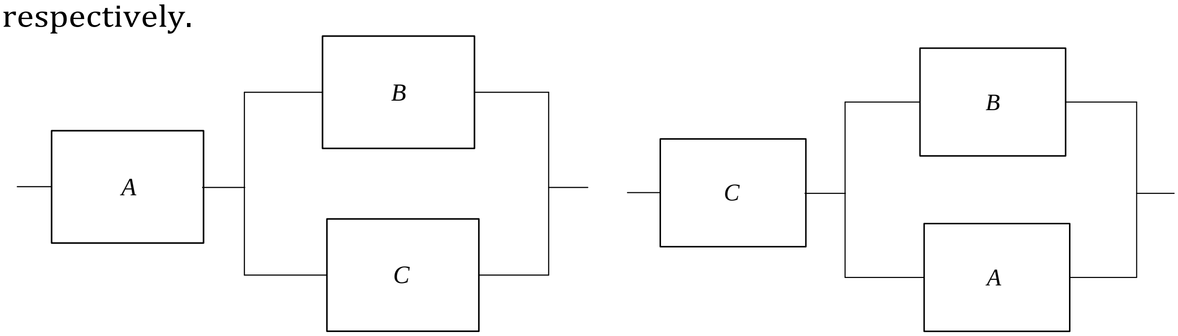
(v) Find and sketch the cumulative distribution function $F(x)$. Indicate all the ranges of the variable x from $-\infty$ to $+\infty$ for $F(x)$. Also, you must label the axis.

(vi) Find the probability $P\left[|X-0.25|\le 0.5\right]$

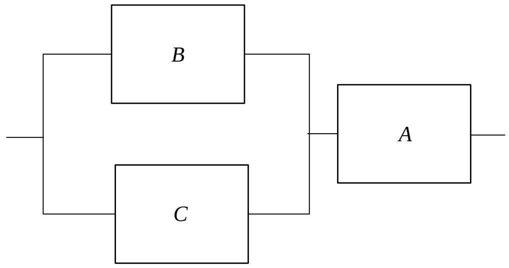
(vii) Suppose the voltage, X, is applied to the input of a nonlinear device with input/output characteristic $Y=X^2+X+1$. What is the average value of the output voltage, Y?

(viii) What is the standard deviation of the voltage, X?

Q.4 The block diagrams below represent series-parallel connections of three subsystems, (A, B and C). The reliability of a subsystem is defined as the probability that it performs correctly under specified conditions. For the given subsystems, the reliabilities are defined as $R_1=P[A]$, $R_2=P[B]$ and $R_3=P[C]$, respectively.



(a) (b)



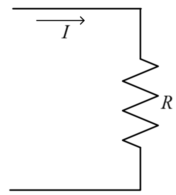
(c)
Figure 1

(i)	Show that, in general, the overall reliability for the system in Figure 1(a) can be expressed as $R_s = P[A \cap B] + P[A \cap C] - P[A \cap B \cap C]$.
(ii)	Given that the operations of the three subsystems in Figure 1(a) are completely independent of each other, express the reliability, R_s, of the overall system, in terms of R_1, R_2 and R_3.
(iii)	What is the overall reliability for the situation in part (ii) above, if $R_1 = 0.8$, $R_2 = 0.75$ and $R_3 = 0.2$?
(iv)	Suppose we now switch the positions of subsystems A and C as shown in Figure 1(b). What is the new overall reliability, for the same subsystem reliabilities as in part (iii), that is, $R_1 = 0.8$, $R_2 = 0.75$ and $R_3 = 0.2$?
(v)	Which is the more reliable configuration, Figure 1(a) or Figure 1(b) and why?
(vi)	Would the reliability improve or worsen if block A is moved to the right side of the parallel blocks B and C as shown in Figure 1(c)? Why or why not?

Q.5

	You are monitoring 3 consecutive packets going through an internet router and classifying each packet as either video (v) or data (d). The outcomes vvv and ddd each have probability 0.2 while the other outcomes vvd, vdv, vdd, dvv, dvd and ddv each have probability 0.1. Let N_v denote the number of video packets in each outcome you observe. Calculate the following probabilities:
(a)	$P[N_v = 2]$
(b)	$P[N_v \geq 1]$
(c)	$P[\{vvd\} \mid N_v = 2]$
(d)	$P[\{ddv\} \mid N_v = 2]$
(e)	$P[N_v = 2 \mid N_v \geq 1]$
(f)	$P[N_v \geq 1 \mid N_v = 2]$

Q.6

(a)	A random current $X = I$ flows through a resistor with $R = 50\ \Omega$. The probability density function (PDF) for the current is given as $f(x) = \begin{cases} 2cx, & 0 \le x \le 0.5 \\ 2c(1-x), & 0.5 \le x \le 1 \end{cases}$ 
(i)	Sketch $f(x)$ and find the value of c which makes $f(x)$ a valid PDF.
(ii)	Find the expected value μ_I of the current I.
(iii)	Find the expected value μ_P of the power dissipated over the resistor.
(iv)	Does the relationship $\mu_P = \mu_I^2 R$ hold for average power?
(b)	We wish to find the mean and variance of a random variable X but all we have are the following expectations: $E\left[(x-1)^2\right] = 10$ and $E\left[(x-2)^2\right] = 6$.
(i)	Find the mean μ of X.
(ii)	Find the variance σ^2 of X

Q.7

(a) The internet connection speed in the ECE Department (U of C) is a random variable X measured in megabits per second (Mbps). The PDF and the true mean of X are both unknown. The true standard deviation is 2 Mbps. We measure the speed on 16 independent occasions and obtain the following results:

4.4	5.5	4.1	5.1	4.2	2.6	4.1	3.8
3.9	4.3	5.1	1.9	2.9	4.9	4.7	4.0

Assume that the sample size is large enough to obtain fairly accurate minimum variance and unbiased estimates of the statistics of X.

Determine an <u>APPROXIMATE PROBABILITY</u> value for $P[3 \leq X \leq 4]$.

State the assumptions that led you to obtain the result.

(b) A battery manufacturer claims that the mean lifetime of their car batteries is 3 years. A battery retail company wants to verify this claim at a significance level of $\alpha = 0.02$. They randomly pick 15 batteries and find their lifetimes (in years) to be given by the following table:

1.9	2.4	3.0	3.5	4.2	2.9	2.5	2.1	2.7	2.8	3.9	1.5	2.0	2.3
1.8													

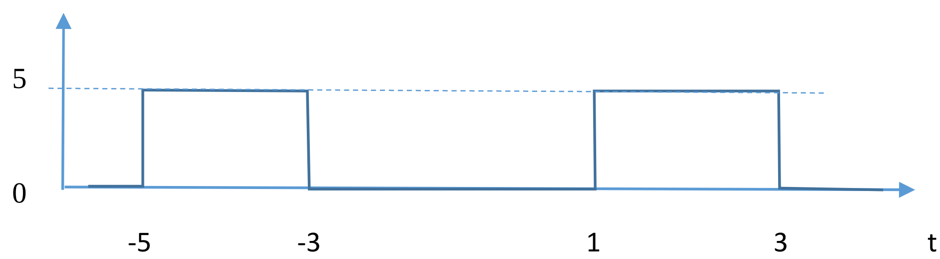
Assume the battery lifetime has a Gaussian distribution.

(i) Construct a hypothesis test that can be used to test the manufacturer's claim. Show all the steps.

(ii) Should the battery retail company reject or accept the manufacturer's claim? Explain why or why not?

TOPIC: Fourier transform, AM/FM modulation (ENEL471)

Q.1 Consider the time domain signal waveform g(t), given in the figure below:



A. Determine the Fourier transform of this waveform

B. Determine the Laplace transform of this waveform

C. Under what conditions or constraints are the Fourier and Laplace transforms the same.

D. Suppose that g(t) is used to AM modulate a carrier signal at a carrier frequency of f_c resulting in

$$y(t) = g(t)\cos\left(2\pi f_c t\right)$$

Sketch the approximate spectrum of y(t) for $f_c = 20$. Annotate your sketch with the frequency and amplitude of the main spectral lobes.

E. Now suppose that g(t) is used to FM modulate a carrier such that the instantaneous frequency is $100 + 4g(t)$ Hz. Determine the phase of the FM modulated signal as a function of time relative to the unmodulated carrier at 100 Hz. Show your results in a sketch.

TOPIC: Wireless communication systems (ENEL529)

Q.1

a) State 2 limitations of frequency reuse. As a wireless system designer, how will you overcome the stated limitations?

b) Mathematically, show that the theoretical maximum number of users that can be supported in a given service area is inversely proportional to *K*, the frequency reuse plan.

c) Consider a cellular system in which there are a total of 490 traffic channels. Suppose the area of a cell is 6 km^2 and the area of the entire system is 2100 km^2. Calculate the maximum number of users that can be supported by the system if the cluster size is 7. Given that the traffic load per user is 0.03 Erlang, the average number of calls per hour per user is 1.5 and, designing for a grade of service of 2% blocking, determine:

i) the offered traffic load per cell in Erlang.

ii) the number of users per km^2 supported by the system

iii) the average duration of a call in seconds

Q.2

a) What are the fundamental ideas behind the frequency division, time division and code division multiple access techniques?

b) Consider the uplink transmission of voice signals in a single-cell direct sequence code division multiple access (DS-CDMA) system with a bandwidth of W Hz. The data transmission rate for each user is R_b bps. Assume that the additive background thermal noise is negligible. To provide satisfactory service quality, it is required that the received signal energy per bit to interference density ratio (E_b/I_o) should be at least $(E_b/I_o)_{th}$. Assuming perfect power control and use of omni-directional antenna in the cell, derive the expression for radio capacity of the CDMA system.

c) Given W = 1.25 MHz, R_b = 9600 bps and $(E_b/I_o)_{th}$ = 7 dB, use the expression derived in b) to calculate the radio capacity for the single-cell DS-CDMA system.

Q.3

a) Explain the similarity and differences between small-scale fading and large-scale fading. Why is shadowing referred to as lognormal?

b) Given that the probability density function of P, the received power in a Rayleigh distributed envelope is given by $f_P(p) = \frac{1}{2\sigma^2}\exp\left[-\frac{p}{2\sigma^2}\right]$ where σ is the parameter, derive the expression for the outage probability, $P_{out} = Prob\{P < P_{\min}\}$, where $P_{\min}$ is the required minimum received power for acceptable communication.

c) For the Rayleigh distributed envelope, given that the average received power $P_{av} = 0.05\ mW$ and $P_{\min}$ is set at 20 dB below the average received power, use the expression derived in part b) to calculate the outage probability. What is the value of $P_{\min}$ in milliWatts?

TOPIC: Digital communication (ENEL571)

Q.1

	A multimedia (video plus audio) signal has a bandwidth of 4.5 MHz. The signal has peak values of $\pm 0.5\text{V}$ while the average power is 20mW. The signal is sampled, quantized and binary encoded to obtain a PCM signal.
(a)	If the signal-to-quantization error (SQNR) is to be at least 43dB, determine the minimum number of bits required to encode the uniform quantizer output.
(b)	(1) What is the Nyquist sampling rate? (2) In order to avoid aliasing the signal is sampled at 25% above the Nyquist rate. What is the new sampling rate?
(c)	(1) What is the minimum transmission rate of the PCM signal if aliasing is to be avoided? (2) What is the bandwidth required to transmit the PCM signal if aliasing is avoided?
(d)	Suppose the PCM signal is to be transmitted over a channel of bandwidth equal to 20 MHz. The sampling rate is the one that does cause aliasing. What is the maximum number of bits required for encoding?
(e)	What is the new signal-to-quantization noise ratio (SQNR) for the conditions in part (d) above?

Q.2

	Consider the binary signal constellation diagram shown in Figure 1. Signal point $\mathbf{s}_1$ represents binary 1 while signal point $\mathbf{s}_2$ represents binary 0. The signal is assumed to be transmitted over an (ideal) AWGN channel. The noise spectral density level is $N_0/2$ Watts/Hz. Denote the noise waveform as $n(t)$.
(a)	Write the two expressions of the transmitted baseband signals for $\mathbf{s}_1$ and $\mathbf{s}_2$, respectively.
(b)	Using any detection procedure of your choice, sketch the block diagram of an optimal receiver (demodulator and detector) for recovering the data bits for the binary signal constellation in Figure 1. Label all the blocks and specify the decision rules for binary 1 and for binary 0.
(c)	Write the expression for the probability of error in terms of E_b and N_0
(d)	Suppose the signal constellation is rotated so that the two signal points lie on the horizontal axis, that is, along $\phi_1(t)$. Write the expression for the resulting probability of error in terms of E_b and N_0
(e)	In general, does rotation of the signal vector (constellation) through an arbitrary angle affect the probability of error (BER) performance? Why or why no?
(f)	In general, does scaling (shrinking or stretching) of the signal vector affect the probability of error (BER) performance? Why or why not?

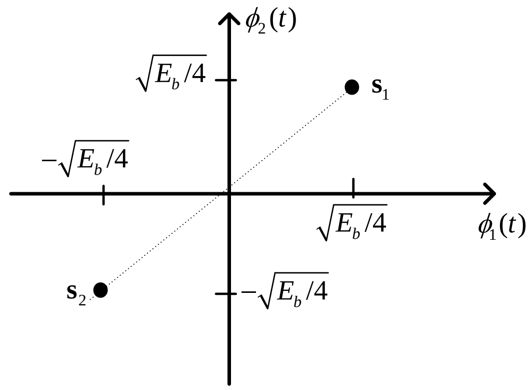


Figure 1

Q.3

(a)	Show whether the following codes are linear: $\mathbf{c}_1 = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0) \qquad \mathbf{c}_3 = (0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1)$ $\mathbf{c}_2 = (0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0) \qquad \mathbf{c}_4 = (0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1)$
(b)	For the parity matrix, $\mathbf{P}$, given below, (i) What is the generator matrix $\mathbf{G}$? (ii) What is the code sequence for input data bit sequence 1101? (iii) What is the parity check matrix $\mathbf{H}^T$?
(c)	Consider a $(7,4)$ Hamming code (code book) given by Table 1. (i) What is the minimum Hamming distance of this code? (ii) What is the error detection capability (that is, the # of detectable errors) of this code? (iii) What is the error correction capability (that is, the # of correctable errors) of this code? (iv) Suppose a code $\mathbf{r} = [1101001]$ is received. Using (Hamming) minimum distance decoding, what code word is the received code closest to?
(d)	For the same received code as in (b), that is, $\mathbf{r} = [1101001]$, (i) Compute the syndrome vector. (ii) Is there an error? Explain why or why not. (iii) If there is an error, which bit is in error? (iv) What is the correct code that was transmitted? (v) Is this code a valid code? (vi) Comparing the results obtained in part (c) (iv) and part (d) (iv), which decoding method yields a more accurate result, the syndrome decoder or the minimum distance decoder?

Table 1: Code books for the (7, 4) Hamming code

$\mathbf{c}_1 = (0\ 0\ 0\ 0\ 0\ 0\ 0)$	$\mathbf{c}_7 = (0\ 1\ 1\ 0\ 1\ 0\ 0)$	$\mathbf{c}_{13} = (1\ 1\ 0\ 0\ 1\ 0\ 1)$
$\mathbf{c}_2 = (0\ 0\ 0\ 1\ 1\ 0\ 1)$	$\mathbf{c}_8 = (0\ 1\ 1\ 1\ 0\ 0\ 1)$	$\mathbf{c}_{14} = (1\ 1\ 0\ 1\ 0\ 0\ 0)$
$\mathbf{c}_3 = (0\ 0\ 1\ 0\ 1\ 1\ 1)$	$\mathbf{c}_9 = (1\ 0\ 0\ 0\ 1\ 1\ 0)$	$\mathbf{c}_{15} = (1\ 1\ 1\ 0\ 0\ 1\ 0)$
$\mathbf{c}_4 = (0\ 0\ 1\ 1\ 0\ 1\ 0)$	$\mathbf{c}_{10} = (1\ 0\ 0\ 1\ 0\ 1\ 1)$	$\mathbf{c}_{16} = (1\ 1\ 1\ 1\ 1\ 1\ 1)$
$\mathbf{c}_5 = (0\ 1\ 0\ 0\ 0\ 1\ 1)$	$\mathbf{c}_{11} = (1\ 0\ 1\ 0\ 0\ 0\ 1)$	
$\mathbf{c}_6 = (0\ 1\ 0\ 1\ 1\ 1\ 0)$	$\mathbf{c}_{12} = (1\ 0\ 1\ 1\ 1\ 0\ 0)$	

Q.4

	Consider the modulated signal $$s_m(t) = \begin{cases} \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + m\frac{2\pi}{M}\right); & 0 \le t \le T_s \\ 0; & \text{elsewhere} \end{cases}$$ where $m = 1, 2, 3, \ldots, M$, T_s the symbol period and E_s the energy per symbol.
(a)	What type of modulation does $s_m(t)$ represent?
(b)	(i) Determine the basis waveforms for $s_m(t)$. (ii) Determine the signal vectors for $s_m(t)$. (iii) Sketch the signal constellation for $M = 4$. Label the axis and signal points. (iv) What is the dimensionality of the signal space?
(c)	Find the minimum distance for the signal points in the constellation above.
	Find the average ***energy per symbol*** and the average ***energy per bit*** for the signal constellation in part (b), that is, for $M = 4$.
(d)	Using the minimum distance obtained in part (c), (i) What is the expression for the probability of ***symbol error***? (ii) What is the expression for the probability of ***bit error***? List all the assumptions you make in arriving at the expression.
(e)	Suppose a client wants you to design a system with the following requirements: symbol error rate $P_{\epsilon s} \le 1.43325 \times 10^{-7}$, symbol transmission rate $R_s = 1.0$ Kbits/Sec and $N_0 = 10^{-1}$ Watts/Hz. Calculate the transmission power per symbol P_s, required to meet those requirements.

TOPIC: Communication networks (ENEL573)

Q.1 You're downloading a 4 Mbyte file over a 2 Mbit/s 802.11b link that uses Stop and Wait ARQ. The system has the following parameters:

- Number of data bits in an information frame: 10000 bits
- Number of overhead bits in an information frame: 208 bits
- Size of an acknowledgement: 112 bits
- Propagation delay between transmitter and receiver: 1 μs
- Processing time required before transmission of next frame: 64 μs.

a) How long will it take to download this file on an error free link?

b) How long will it take to download this file on a link where the probability of bit error is 0.0001?

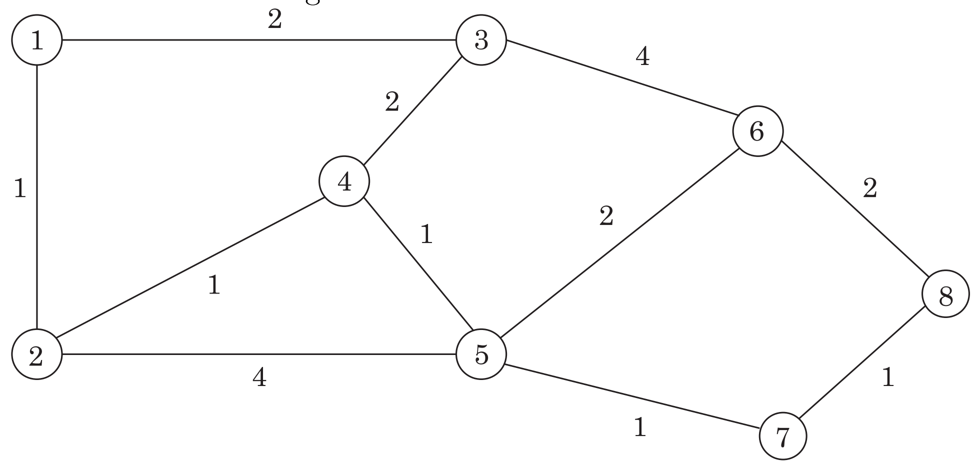
Q.2 Assume a system with 4000 bit frames, a data rate of 2 Mbit/s and an ALOHA MAC. New frames arrive in the channel according to a Poisson distribution.

a) For a frame arrival rate of 2 per frame duration, determine the probability that exactly one frame collides with our desired frame.

b) For frame arrival rates of 2 and 4 per frame duration, determine the probability of 1 or more frames colliding with our desired frame.

c) Determine the effective throughput of the channel in *bits/second* when the frame arrival rate is 2 and 4 per frame duration.

Q.3 The following graph represents a multi-hop network. The circles represent computer and the edges represent network links between each computer. Each link is assigned a number which indicates the cost of transmitting over that link.



Consider a scenario where node 1 wants to send a packet to node 8 and the paths through the network are determined using the Bellman-Ford algorithm.

a) Using a table that contains a row for each iteration, show how the Bellman-Ford algorithm determines the minimum path from node 1 to node 8. Your table should contain the next hop to node 8 routing table entries for all nodes in the network. At what iteration does it have a path to node 8 and at what iteration is that path the minimum cost path?

b) Assume now that the link between nodes 2 and 4 goes down. By adding more rows to the table you created for part a), show how the Bellman-Ford algorithm compensates for this. What is the new path between nodes 1 and 8 and how many iterations does it take for the algorithm to find that path?

Q.4 Consider a router where packets arrive at a rate of 500,000 packets/sec and the size of the packets is exponentially distributed with an average size of 230 bytes. The router has a single output link that runs at 1 Gbps.

a) Determine the average number of packets in the router buffer and the average time a packets spends in the buffer if we assume the router buffer size is infinite.

b) Determine the average number of packets in the router buffer if the buffer size is equal to 50 packets. Is this number larger or smaller than what you calculated for part a)? Explain why.

c) What's the smallest buffer size, in bytes, that we can have and ensure a probability of overflow of no more than 10%?

TOPIC: z-transforms, digital filters (ENEL593)

Q.1 For the linear time-invariant (LTI) system shown in Fig. 1.

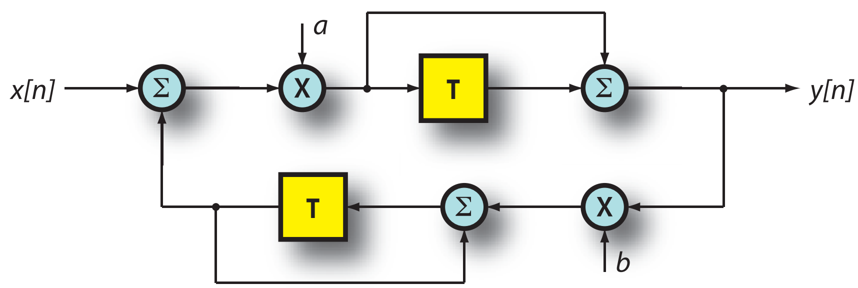


Fig. 1. Block diagram of a discrete-time system

(a) Assuming zero initial conditions, determine the z-domain transfer function $H(z) = Y(z)/X(z)$.

(b) Sketch the *transpose* of this network.

Q.2 Let there be an LTI system with $H(z)$ given by:

$$H(z) = \frac{z+1}{z-0.5}. \tag{1}$$

(a) Show that the difference equation for this system is:

$$y[n] = x[n] + x[n-1] + 0.5y[n-1]. \tag{2}$$

(b) Using the unilateral z-transform, determine and sketch the response $y[n]$ of this system when the input is the unit step sequence $x[n] = u[n]$, *with the following initial condition:*

$$y[-1] = -2$$

(c) Give the region of convergence of the z-transform for $Y(z)$.

Table of z-Transform Pairs

	$f[k]$	$F(z)$
1	$\delta[k-j]$	z^{-j}
2	$u[k]$	$\frac{z}{z-1}$
3	$ku[k]$	$\frac{z}{(z-1)^2}$
4	$k^2u[k]$	$\frac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$	$\frac{1}{z-\gamma}$
7	$\gamma^k u[k]$	$\frac{z}{z-\gamma}$
8	$k\gamma^k u[k]$	$\frac{\gamma z}{(z-\gamma)^2}$
9	$k^2\gamma^k u[k]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\frac{k(k-1)(k-2)\ldots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$\lvert\gamma\rvert^k \cos(\beta k)u[k]$	$\frac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b	$\lvert\gamma\rvert^k \sin(\beta k)u[k]$	$\frac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$	$\frac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$	$\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}, \quad \gamma=\lvert\gamma\rvert e^{j\beta}$
12c	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$	$\frac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

$$r=\sqrt{\frac{A^2|\gamma|^2+B^2-2AaB}{|\gamma|^2-a^2}}$$

$$\beta=\cos^{-1}\frac{-a}{|\gamma|}, \quad \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{|\gamma|^2-a^2}}$$

Field of Study Examination, Feb 24 2017

Subject area: Telecommunications, Signal Processing and Image Processing

This question paper has 7 pages (not including this cover page).

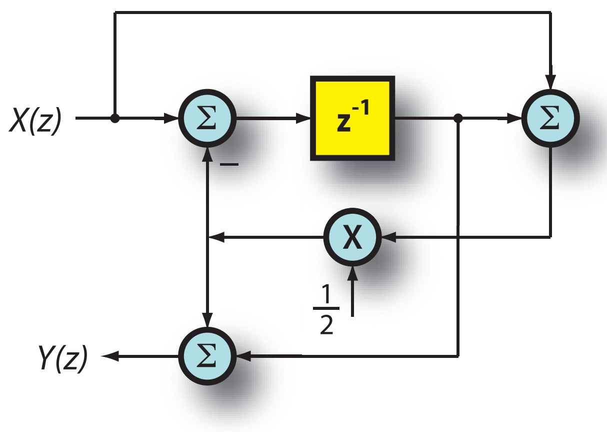
This question paper has 6 questions.

Answer a minimum of one question and at most three questions from this subject area.

Use a separate booklet (i.e., blue booklet) for the answers to questions in this subject area.

1. This question has two parts (a)-(b). Part (a) has 4 subparts (i)-(iv).

   (a) Consider the z-domain block diagram of a LTI system shown below. Assume that initial conditions are zero. Use the z-transform in the following calculations.



      i. Prove or disprove that the z-transform transfer function $H(z)$ is given by

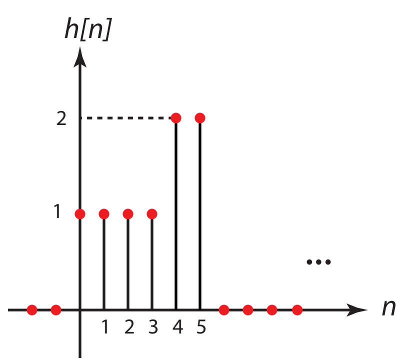
$$H(z) = \frac{\frac{1}{2}z + 1}{z + \frac{1}{2}}. \tag{1}$$

      ii. Using the equation (1), determine the response $y[n]$ of this system when the input is the unit step sequence $x[n] = u[n]$.

      iii. Give the input-output difference equation corresponding to equation (1).

      iv. Briefly describe the steps involved in solving for the step response of the system *with* an initial condition $y[-1] \neq 0$.

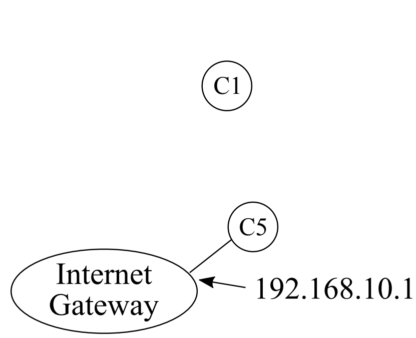
   (b) Consider a LTI system with the unit impulse response $h[n]$ given as shown below.



   Using discrete-time convolution, determine and carefully sketch the output $y[n]$ of the system if the input is given by $x[n] = u[n-4]$.

2. This question has two parts (a)-(b). Part (a) has 2 subparts (i)-(ii) and part (b) has 3 subparts (i)-(iii).

   (a) In the following network diagram, nodes C1, C3 and C2 each have one available ethernet port, node C4 has two available ethernet ports and node C5 has three available ethernet ports. Node C5 also has a fourth ethernet port connected to a computer that acts as a gateway to the Internet. You also have an unlimited number of ethernet switches available to you **but** these switches have only two ports.



   Answer the following questions.

   i. Draw a network that uses *only* the two port switches and *only* the available ethernet ports in each node to connect all five nodes so that they can communicate with each other and the Internet. You are not allowed to add extra ports to the nodes or to the switches. Switches should be drawn as squares with the letter "S" inside them.

   ii. Assume that you are populating the routing tables in your network using the Bellman-Ford algorithm. How many iterations of that algorithm would it take before your routing tables stabilized? You can assume that the links between each node in your network have a cost of 1. Show your work.

   (b) You are analyzing a stop and wait ARQ scheme with the following parameters:

   - Length of ACK Frame: 100 bits
   - Propagation Delay: 3 μs
   - Processing Delay: 20 μs
   - Raw Link Throughput: 2 Mbit/s
   - Overhead in Information Frame: 20 bits

   Answer the following questions:

   i. Assuming an error free channel, determine the number of information bits per frame required to achieve an efficiency of 80%.

   ii. Now assume the channel is noisy. Using your answer from part (i), determine the bit error rate if each frame requires an average of 1.5 retransmissions.

   iii. What is the effective throughput experienced by the user for the error rate you calculated in (ii)?

3. This question has 2 parts (a)-(b). Part (a) has 4 subparts (i)-(iv) and part (b) has 3 subparts (i)-(iii).

Consider a multi-level modulated waveform given by

$$\begin{aligned} s_m(t) &= \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t - \frac{\pi}{4} + \frac{2m\pi}{M}\right) \\ m &= 1, 2, ..L.., M = 2^k \end{aligned}$$

where k is number of bits per symbol and $f_c \gg 1/T_s$

(a) i. What type of modulation does this waveform represent?

ii. It is desired to determine the signal space representation for $\{s_m(t)\}_{m=1}^{M}$. Find a set of basis waveforms $\{\phi_l(t)\}_{l=1}^{M}$ for representing $\{s_m(t)\}_{m=1}^{M}$.

iii. Find the signal points or vectors $\{s_m\}_{m=1}^{M}$ for signal waveforms $\{s_m(t)\}_{m=1}^{M}$.

iv. Show that, in general, the minimum distance between adjacent signal points is

$$d_{min} = 2\sqrt{E_s} \sin\left(\frac{\pi}{M}\right)$$

(b) i. Sketch and completely label the signal constellation diagram for $M = 4$

Suppose the signal represented by the above constellation is transmitted over a channel corrupted by an additive white Gaussian noise with a mean of zero and standard deviation $\sigma_n = \sqrt{N_0/2}$. Answer the following questions:

ii. Find and express the average probability of **symbol error** in terms of the Q-function defined below.

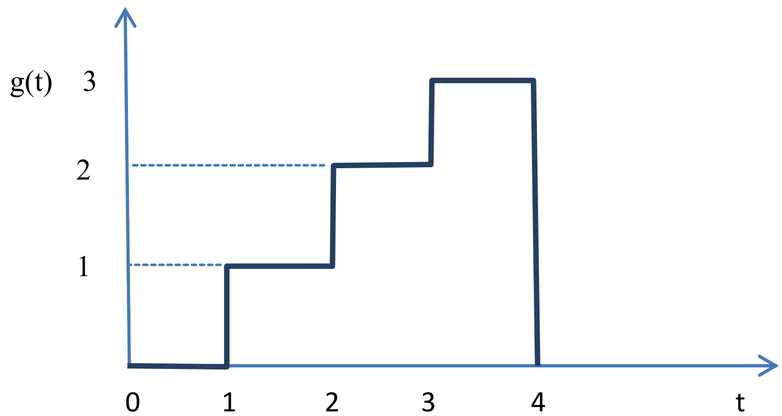
iii. From the above result, write the expression for the probability of **bit error**.

Assume that all signal points are equally likely to occur and only adjacent points can cause errors.

$$\textbf{Definition: } \text{Prob}\left[\|s_i - s_j\| > \frac{d}{2}\right] = Q\left(\frac{d}{2\sigma_n}\right)$$

4. This question has 2 parts (a)-(b). Part (a) has 3 subparts (i)-(iii) and part (b) has 2 subparts (i)-(ii).

   (a) Consider the time domain function shown in the figure below



      i. Express the waveform $g(t)$ in the plot as a function of pulse functions
      ii. Determine the Fourier transform $G(f)$, of the function $g(t)$ as a sum of sinc(t) functions where
$$\mathrm{sinc}(\mathrm{t}) = \frac{\sin(\pi \mathrm{t})}{\pi \mathrm{t}}$$
      iii. Determine which frequencies satisfy $G(f) = 0$ for $|f| < 2$

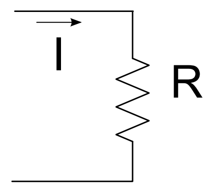
   (b) Consider an AM modulated signal of the form
$$g(t) = (1 + m(t)) \cos(2\pi f_c t)$$
   where f_c is the carrier frequency and $m(t)$ is the modulation signal with a Fourier transform of $M(f)$.

      i. What is the Fourier transform of $g(t)$ which is denoted as $G(f)$?
      ii. The AM signal passes through a nonlinear amplifier that is characterised as follows. If a signal of $x(t)$ is input into the amplifier then the output is $y(t) = Ax(t) + Bx(t)^2$ where A and B are positive real constants. Determine the Fourier transform of the output of the amplifier resulting from an input of $g(t)$.

5. This question has 2 parts (a)-(b). Part (a) has 5 subparts (i)-(v) and part (b) has 3 subparts (i)-(iii).

(a) A random current $I = X$ amps flows through a resistor with $R = 50\ \Omega$. The probability density function (PDF) for the current is given as

$$f_X(x) = \begin{cases} cx & 0 \le x \le 1 \\ c(2-x) & 1 < x \le 2 \end{cases}$$



i. Sketch the pdf, $f_X(x)$, and find the value of the constant c

ii. What is the expected value, μ_X, of the current?

iii. What is the expected value, μ_P, of the power dissipated in the resistor?

iv. We could also compute average power in terms of the mean of the current as $\mu_P = \mu_X^2 R$. Is this result the same as that obtained in part (iii)? Explain why they are equal or why they are not equal.

v. Find the standard deviation of the current.

(b) The random variable X is described by the following equations

$$\begin{aligned} E\left[(x-1)^2\right] &= 10 \\ E\left[(x-2)^2\right] &= 6 \end{aligned}$$

i. Find the mean of X.

ii. Find the mean-square value of X.

iii. Find the standard deviation of X.

6. This question has 3 parts (a)-(c). Part (c) has 4 subparts (i)-(iv).

(a) Explain how the frequency reuse plan K presents a tradeoff between the system capacity and transmission quality in a cellular system.

(b) Assume a shadow fading environment in which the receiver sensitivity is P_{min}. The outage probability P_{out} is the probability that the received power falls below P_{min} given by: $P_{out} = Q\left(\left(P_{av} - P_{min}\right)/\sigma_{dB}\right)$ where P_{av} is the average received power. Derive a closed-form expression for $M_{sh.dB}$ the required shadow fading margin, expressed in dB, such that an outage probability objective $P_{out,obj}$ is not violated.

(c) Consider two cellular systems that have the following characteristics. The total available spectrum W is 20 MHz and the channel bandwidth W_{ch} is 30 kHz. The two systems are distinguished by the frequency reuse plan K, which is 4 and 7, respectively.

i. Suppose that in each of the two systems, the cluster of cells (K=4, K=7) is duplicated 16 times. Find the theoretical maximum number of simultaneous users that can be served by each system.

ii. Find the number of simultaneous users that can be served by a single cell in each system.

iii. How many cells are in the service area of each system?

iv. Suppose the cell size is the same in the two systems and a fixed service area of 100 cells is covered by each system. Find the maximum number of simultaneous users that can be served by each system.

Table of z-Transform Pairs

	$f[k]$	$F(z)$
1	$\delta[k-j]$	z^{-j}
2	$u[k]$	$\frac{z}{z-1}$
3	$ku[k]$	$\frac{z}{(z-1)^2}$
4	$k^2u[k]$	$\frac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$	$\frac{1}{z-\gamma}$
7	$\gamma^k u[k]$	$\frac{z}{z-\gamma}$
8	$k\gamma^k u[k]$	$\frac{\gamma z}{(z-\gamma)^2}$
9	$k^2\gamma^k u[k]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\frac{k(k-1)(k-2)...(k-m+1)}{\gamma^m m!}\gamma^k u[k]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$\|\gamma\|^k \cos(\beta k)u[k]$	$\frac{z(z-\|\gamma\|\cos\beta)}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
11b	$\|\gamma\|^k \sin(\beta k)u[k]$	$\frac{z\|\gamma\|\sin\beta}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
12a	$r\|\gamma\|^k \cos(\beta k+\theta)u[k]$	$\frac{rz[z\cos\theta-\|\gamma\|\cos(\beta-\theta)]}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
12b	$r\|\gamma\|^k \cos(\beta k+\theta)u[k]$	$\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}, \quad \gamma=\|\gamma\|e^{j\beta}$
12c	$r\|\gamma\|^k \cos(\beta k+\theta)u[k]$	$\frac{z(Az+B)}{z^2+2az+\|\gamma\|^2}$

$$r=\sqrt{\frac{A^2|\gamma|^2+B^2-2AaB}{|\gamma|^2-a^2}}$$

$$\beta=\cos^{-1}\frac{-a}{|\gamma|}, \quad \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{|\gamma|^2-a^2}}$$